https://doi.org/10.1051/e3sconf/202336803010

Summary of the Soil Reinforcement Technical Committee Special Session (IGS TC-R)

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Abstract. This document provides a summary of the different topics presented at the Special Session organized by the International Geosynthetics Society (IGS) Technical Committee on Soil Reinforcement (TC-R). This Special Session brings together very interesting studies regarding soil reinforcement in the field of geosynthetics. Studies presented include topics both from theoretical and practical points of view of reinforcement geosynthetics including general products and applications, cases studies on road embankments under challenging site boundary conditions, research on deterministic and probabilistic design of reinforced fills over voids, numerical analysis of reinforced soil wall structures, encased granular column technique, and geosynthetic-reinforced bridge abutment behavior.

1 Background and IGS TC-R introduction

Polymeric reinforcement materials are currently used routinely as soil reinforcement and stabilization, as well as for barrier systems (waterproofing) or with hydraulic functions (drainage) in the framework of Civil Engineering [1]. Fibrous filter fabric products were already used in the ancient times to improve the mechanical performance of the soil (using

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natural fabrics or vegetation), but it is in the last third of the 20th century when, after the advent of polymeric materials, more stable and durable products were available; then, technology advanced, their effectiveness increased significantly, and previously unknown or unconsidered applications became relevant. The list of the most commonly used geosynthetic products currently available is extensive (geotextiles, geogrids, geonets, geomembranes, geocells, geocomposites, etc.), which differ in the constitutive material (most are comprised of polyamide, polyester, polyethylene, polypropylene, polyvinylchloride, ethylene copolymer, etc.), product shape (sheets, grids, cells, straps, etc.), and consequent performance and design related to the required function [1, 2]. Geosynthetics can play an important role in meeting the global challenges facing society in terms of the United Nations sustainability goals in both mitigating and adapting to the impacts of climate change [3, 4]. With current and future improvements on the effects of human activity on climate change and global warming. Thus, it has never been more important to share knowledge and strategies to tackle this global problem efficiently.

The International Geosynthetics Society (IGS, https://www.geosyntheticssociety.org/) is a global learned society dedicated to the scientific and engineering development of geosynthetic materials and associated technologies. Represented by engineers, researchers and technical staff around the world, the core purpose of the IGS and its Technical Committees (TC-Barrier Systems, TC-Hydraulics, TC-Soil Reinforcement, and TC-Stabilization, formed between 2010 and 2018) is to provide the understanding of and to promote the appropriate use of geosynthetic technologies throughout the world, as well as establishing strong templates for participation and influence within the IGS and in the larger Geotechnical Engineering community. The Technical Committee on Soil Reinforcement (TC-R) is dedicated to the scientific and engineering development of geosynthetic systems and associated technologies. This committee promotes the dissemination of knowledge, technology, research findings, design and construction methodologies related to reinforcement applications using geosynthetics, such as: reinforced soil walls and abutments; reinforced soil steep slopes; basal reinforcement of embankments on soft soil, on piles, and on voids; and, veneer reinforcement of shallow soil layers on slopes.

This document provides a summary of the different topics presented at the Special Session organized by the International Geosynthetics Society (IGS) Technical Committee on Soil Reinforcement (TC-R), chaired by Ivan P. Damians and Pietro Rimoldi (current Secretary and Chair of the IGS TC-R, respectively). This Special Session brings together very interesting studies regarding geosynthetic-based soil reinforcement function in the field of Civil Engineering. The document is comprised of extended abstracts submitted by the presenters, and arranged in the presentation order of the speakers at this special session. Studies presented include a range of topics both from theoretical and practical points of view of reinforcement geosynthetic applications, which are briefly summarized as follows, and with more details included in the further sections.

2 Summary of the presented topics at the TC-R Special Session

- *Geosynthetics for reinforcement - products and applications* (presented by Pietro Rimoldi and co-authored by Y. Miyata and I.P. Damians):

In this presentation, the main families of geosynthetics for soil reinforcement function are introduced, as well as the main related applications in the Geotechnical Engineering field, which are: reinforced soil walls and slopes, basal reinforcement of embankments (over soft soil, over piles, over voids), and reinforced soil veneer. Several pictures and details about representative structures, related components and important projects from worldwide are shown to illustrate the actual and potential applications of geosynthetic-based soil reinforcement in the Geotechnical Engineering field.

- *Geotechnical challenges in large-scale infrastructure projects in Africa* (presented by Oliver Detert and co-authored by S. Uelzmann, M. Hoelzel, A. Kirchner):

In the south of Nigeria, within the Niger Delta, which is up to 200 km wide, transport infrastructure measures pose a particular challenge due to the low load-bearing soils, the jungle-like terrain that is difficult to access, the large number of large rivers and small watercourses that have to be crossed, and the large seasonal fluctuations in the water level of up to 10 m. The construction of road embankments is also a major challenge. This applies in particular to road embankments, as additional construction measures are usually required to ensure their stability and serviceability, and these measures must be adapted to the specific local conditions. The presentation is reporting two major road construction projects currently under construction and their execution under very challenging boundary conditions.

- *Deterministic and probabilistic design of reinforced fills over voids* (presented by Richard J. Bathurst and co-authored by F. M. Naftchali):

The presentation provides a brief overview of recent research by the authors that focuses on the deterministic and probabilistic design of reinforced fills over voids. The work introduces a new geosynthetic stiffness limit state in addition to deformation, tensile strength and tensile strain limit states found in the literature. A simple two-parameter hyperbolic isochronous load-strain model is used to characterize the stiffness of the reinforcement as a function of elapsed time and strain. For probabilistic analyses, the uncertainty in the estimate of the reinforcement stiffness using the hyperbolic model is used. A practical outcome of this work is a quantitative link between the nominal factor of safety used in deterministic working stress design practice and reliability index; the latter is used in contemporary performance-based design to quantify margins of safety within a probabilistic framework.

- *Numerical analysis of reinforced soil wall structures* (presented by Ivan P. Damians):

Through the development of numerical models it is possible to better understand the mechanical behavior of soil structures reinforced with polymeric reinforcement. The given presentation and related cited references present numerical analysis of reinforced soil wall structures including both comparisons between different numerical software and calculation methodologies, results obtained in both 2D and 3D representations, as well as real project case study examples. Based on previously calibrated models, it has been studied, in particular, the connections of the reinforcements with the facing, the soil-structure interactions (soil-reinforcement and soil-facing), the stress state in the wall facing through the behavior of the support elements involved, the effect of the rigidity of the fill soil and the foundation, the pre-stressing of the reinforcements, and a thermo-hydraulic approach in development. The results presented in this document show, for example, that variations both in the stiffness of the ground and in the characteristics of certain construction elements cause significant changes in the state and in the distribution of stresses and strains in the structure. The cited references provide greater detail regarding the points discussed in the presentation.

- *Behavior and failure mechanisms of geosynthetic encased columns: Lessons learned from recent experiments* (presented by Cihan Cengiz):

Ordinary Stone Columns (OSCs) and Geosynthetic Encased Columns (GECs) are among proven soft soil remediation techniques which typically enhance the economic viability and delivery times of a project. The state of the art on these remediation techniques have vastly been confined to a subset of conceivable loading regimes and associated failure mechanisms which are characterized by static vertical load cases and punching and bulging failures, respectively. Other loading conditions such as lateral loading or dynamic loading components which can cause shear or bending failures of the column are lesser known and rarely investigated in the presently available literature. The presentation attempts to shed light into these aspects of granular column behavior by tapping into a recently conducted pipeline of 1-g physical tests, showcasing some of the lesser investigated response patterns of granular columns. It is believed that sharing these results with the wider geotechnical community will draw attention to the lesser known and possibly more catastrophic failures. It is also hoped that the future designs will consider relevant failure patterns that are demonstrated in this contribution. Interested readers are referred to the cited works of the author for detailed explanations of the behavioral patterns and failure modes where these findings are discussed in detail. The presentation delivered in the conference aims to cover these aspects in depth.

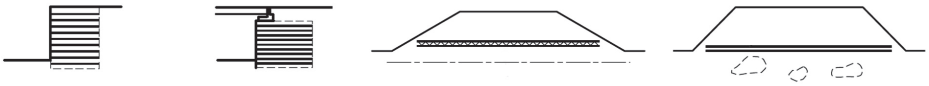
- *Behavior of load-carrying geosynthetic-reinforced bridge abutments* (presented by Jorge G. Zornberg and co-authored by A.M. Morsy):

The technology of geosynthetic-reinforced soil has been used extensively in transportation systems for earth retention and to support vertical loads from the self-weight of the backfill soil, roadway structures, and traffic loads. A comparatively new application of this technology are load-carrying geosynthetic-reinforced bridge abutments, designed to directly support the load of bridge superstructure without the need of deep foundations. They have the potential of alleviating the "bump at the bridge" and expediting construction at reduced financial and carbon costs while, at the same time, requiring comparatively smaller working areas and simple machinery. The presentation delivered provides an overview of the design philosophy and construction techniques of this new technology, which have often differed significantly in worldwide applications.

3 Geosynthetics for reinforcement - products and applications

Soil reinforcement is an ancient concept. Only the advent of geosynthetics allowed to apply the modern concept of soil reinforcement, thanks to the availability of industrialized products with constant and predictable technical characteristics. Over forty years of successful projects and positive experiences, extensive testing and research, development and validation of design methods, have demonstrated the superior performance of reinforced soil structures subjected to all types of permanent and accidental loads; moreover, reinforced soil structures have demonstrated to afford high seismic resistance both in ultimate and serviceability limit states. Reliable design methods and construction procedures are currently available for each required application using soil reinforcement with geosynthetics. Furthermore, reinforced soil structures provide higher sustainability and lower carbon footprint than traditional structures, as well as have shown to be a stable, flexible, resilient, easy to build and construction time saving alternative to traditional techniques in many situations.

Products for soil reinforcement are either steel or polymeric elements, which can be classified according to EN 14475 [5]. Geogrids are characterized by the open structure and high tensile strength. Geogrids can be uniaxial, biaxial, or multiaxial. Geogrids can be divided in three main groups, according to the production technology: extruded geogrids; woven / knitted geogrids; bonded geogrids. The first geogrids were uniaxial, produced by extrusion, cold punching and longitudinal stretching; these geogrids were developed in the U.K. in 1980. Bonded geogrids were developed in the U.K. in the late 1970s, using strips of bundles of polymeric coated polyester fibres, bonded at right angles. Bonded geogrids can be both uniaxial and biaxial. The single strip became the first geostrip. Recently these geogrids have been developed as the first draining geogrid, by shaping the longitudinal straps as channels, each covered by a strip of filtering geotextile. Another group of bonded geogrids are made by laser or ultrasonically bonding together polyester or polypropylene rods or straps in a grid like pattern. This type of geogrid was developed in the Netherlands and in Germany in the early 1990's. The first woven geogrid was developed in Germany in the mid 1980's: these polyester yarn geogrids were made on textile weaving machinery. Almost at the same time in the Netherlands knitted geogrids were developed, made from high tenacity polyester yarns covered with a black polymeric coating. In these two processes hundreds of continuous fibers are gathered together to form yarns that are woven or knitted into longitudinal and transverse ribs with large open spaces between. The crossovers are joined by intertwining or knitting before the entire unit is protected by a subsequent coating. Woven and knitted geogrids can be produced both uniaxial and biaxial. Woven and knitted geotextile have a linear structure, which affords their use mainly for mechanical functions like reinforcement. There are five principal types of fibers used in the production of woven and knitted geotextiles: monofilament, multifilament, staple fiber yarn, slit-film monofilament, and slit-film multifilament. The type of fiber and the type of polymer have influence on the mechanical, hydraulic, and durability properties of the geotextiles. Geosynthetics fulfil the function of reinforcement in the main application areas of reinforced soil walls and slopes, basal reinforcement of embankments (over soft soil, over piles, over voids), and reinforced soil veneer (see schematic layouts in Figure 1).


Reinforced soil walls and abutments
Embankments in soft soils
Embankments over voids

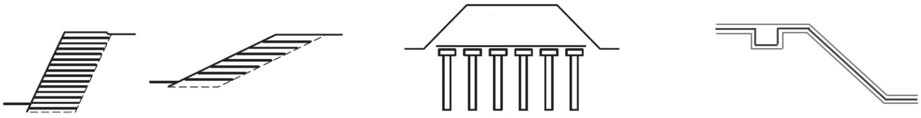

Reinforced slopes
Piled embankments
Veneer reinforcement

Fig. 1. Applications of soil reinforcement (modified from [6]).

3.1 Reinforced soil walls and slopes

Reinforced fill structures are constructed using successive layers of compacted, selected fill incorporating intervening layers of horizontal reinforcement placed at designed spacing. The components of a reinforced fill structure are: the reinforced fill, which includes the reinforcing elements and makes the reinforced block; the retained fill, behind the reinforced block; the natural soil and the foundation soil; the facing system. Reinforced soil walls and abutment are retaining structures with vertical facing, while reinforced slopes have non-vertical facing. Retained earth reinforced soil structures have demonstrated to afford much higher seismic resistance than conventional retaining structures.

Reinforced fill earth retaining structures, with a vertical, battered or inclined face, require a facing to retain the fill between the reinforcing layers. Depending on the particular system, certain layers of fill reinforcements may however not be connected to the facing. The facing can be constituted of either hard units (typically made of concrete), or deformable units (typically made from steel grids or gabion baskets), or soft units (typically made from wrap-around geosynthetics or woven wire meshes).

Where hard or deformable facing units are used, these serve as a formwork against which the selected fill is placed, compacted, and contained. Where soft facing units are used, it is generally necessary to employ temporary formwork to maintain the face alignment during the construction of walls or steep slopes. Hard facing units are usually produced in precast concrete, either unreinforced or reinforced. Concrete facing units may be full height panels, partial height panels, or segmental blocks. Many types of concrete facing units are proprietary and form part of proprietary systems. The reinforcements are connected to the units either by means of devices embedded or inserted into the concrete units, or they are simply clamped between the units. Figure 2 show examples of polymeric straps reinforced fill structures with pre-cast concrete panels. Compatible also with the same type of reinforcement, facing units may be also formed by masonry blocks, and of either closed gabions or open-backed welded wire mesh panels, either flat or pre-bent to the required slope angle, serving as a formwork during construction. Wrap-around facing are widely used soft facing unit, where full width reinforcements, such as geogrids or geotextiles or woven wire meshes, are extended forward from the reinforced fill to wrap around the face of each intervening layer of fill.

Fig. 2. Examples of polymeric strip reinforced soil walls and abutments in Middle East [7].

3.2 Basal reinforcement

In the case of an unreinforced embankment, the stability of the embankment can be ensured by reducing the lateral displacement or the settlement. Basal reinforcement (Figure 3a) is effective to construct high embankment on soft ground: its mechanism is the tensile force and the confinement of soft soil generated by the tensioned membrane effect, which is due to the out-of-plane deformation produced in the geosynthetic by the weight of the embankment.

In case of basal reinforcement with piled embankment (Figure 3b), the mechanisms involved are the arching effect in the soil above piles and the tensile force in the geosynthetic generated by the tensioned membrane effect between adjacent piles.

Progressive formation of voids can cause serious geotechnical damage to infrastructures, like roads and railways. Applying basal reinforcement, the geosynthetic reinforcement deforms across the void and supports the fill in the embankment, then the deformation of the ground surface is maintained below the serviceability limit.

a) b)

Fig. 3. Examples of basal reinforcement: (a) embankment on soft soil and (b) piled-based road embankment [8].

3.3 Reinforced soil veneer

Veneer is intended as a relatively thin cover soil layer placed on a slope. Soil veneer reinforcement is the only reinforcement application where reinforcement is not placed horizontally but sloping (Figure 4). There are two main applications of reinforced soil veneer:

- Leachate collection soil placed above a waterproofing liner along the sides of a landfill or a heap leach pad before waste or ore is placed and stability achieved accordingly.
- Final cover soil placed above a waterproofing liner in the cap or closure of a landfill or a heap leach pad.

As the veneer layer becomes longer and steeper, the tensile strength required for the geosynthetic reinforcement becomes quickly very high, of the order of hundreds kN/m. Such high tensile strength have to be transferred to the veneer layer, hence the interface properties soil-geosynthetic are very important. Below the reinforcing geosynthetic usually there is the lining system, hence interlocking between soil and geogrid becomes very limited, while the friction angle soil-geotextile may be too small for transferring high tensile forces. Hence a reinforced geomat, made up by a tridimensional geomat factory bonded to a high strength geogrid or geotextile, is usually the preferred choice.

a) b)

Fig. 4. Examples of soil veneer with (a) reinforced geomats and (b) uniaxial geogrids.

4 Geotechnical challenges in large-scale infrastructure projects in Africa

Two case studies regarding road embankments in Niger are presented. In both ones, very low bearing capacity of the soil were encountered. Also, particular local boundary conditions made difficult to perform the construction works and even to access to the site.

4.1 Second Niger Bridge Project

The Second River Niger Bridge is a key national infrastructure project with immense benefits for the population and economy of Nigeria. It involves the erection of the 1.6 km long bridge as well as the construction of more than 10 km of highway within the River Niger delta and swamp area (see Figure 5a) in a forecasted 42 month construction period. Due to the very low load-bearing capacity of the subsoil in some areas, combined with very long consolidation times, 17,000 geosynthetic-encased sand columns (GEC) with a total length of 200,000 linear meters were also installed to improve the subsoil.

To further improve the global stability of the embankments, the GEC were installed along with 840,000 m² of geotextiles with tensile strengths of up to 2,500 kN/m as horizontal reinforcement in its footprint (Figure 5b).

Julius Berger Nigeria PLC (JBN) is currently undertaking the project, which, in addition to the river bridge and the road embankments, also includes a toll station, a motorway interchange and two flyover bridges. Julius Berger International GmbH (JBI), a subsidiary of JBN, is responsible for the majority of the design work, consisting of infrastructure planning, structural design, structural engineering, work preparation, formwork and scaffolding, geotechnical engineering, as well as purchasing, export and coordination of external design offices.


a) b)

Fig. 5. Second River Niger Bridge: (a) swamped construction site detail and (b) highway embankment under construction.

4.2 Bodo Bonny Road

In the marshlands of the Niger Delta with countless watercourses and under the influence of the tides, one of the largest gas liquefaction plants in Africa was built on the offshore island of Bonny Island about 30 years ago. Since then, the island was to be connected to the mainland with a road and bridge project - but all previous attempts failed. At the end of 2017, the cooperation between the Nigerian government and the parastatal LNG company awarded the major project to Julius Berger Nigeria (JBN). Julius Berger International (JBI) in Wiesbaden took over the entire planning. After almost two years of construction planning, the scope of the project was finalized: 17 incremental and prefabricated bridges as well as 38 km of road construction with 2.8 million m³ of sand as dam construction material, 570 thousand m² of geosynthetics for dam reinforcement and almost 10 million linear meters of prefabricated vertical drains (PVD) to consolidate the up to 30 m thick soft, water-saturated soils must be completed on schedule.

5 Deterministic and probabilistic design of reinforced fills over voids

Analytical and numerical solutions for the problem of geosynthetic reinforced fills over voids have been the subject of investigation for the last four decades, appearing frequently in the research literature and in design guidance documents. The general problem is illustrated in Figure 6. The voids may develop due to thawing in permafrost, collapse of mine workings, karstic terrain, openings below landfill liners and subsurface pipe failure. A common feature of this prior work is that all methods have treated the analytical solutions as deterministic. The given presentation revisits the problem of geosynthetic reinforced fills over voids for vertical deformation, maximum strain and maximum tensile load limit states and a new geosynthetic stiffness limit state. While the treatment of some input parameters must be taken as deterministic, soil and geosynthetic properties have uncertainty to different degrees. The presentation reviews a body of work by the authors that investigates the three last limit states from both a deterministic and probabilistic point of view.

Some novel features of the work by the authors for the analysis and design of this problem are: (1) adoption of a two-component hyperbolic model to capture the creep-dependent behavior of rate-dependent geosynthetic sheet reinforcement materials used to support the fill over a void; (2) introduction of a new geosynthetic stiffness limit state; (3) a probabilistic design approach that considers uncertainty in the estimate of the reinforcement stiffness for the geosynthetic stiffness limit state, and; (4) a quantitative link between nominal factor of safety used in deterministic working stress design practice and reliability index; the latter is used in contemporary performance-based design to quantify margins of safety within a probabilistic framework.

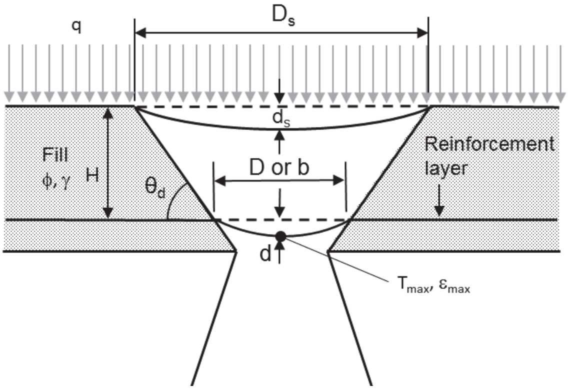


Fig. 6. Reinforced thin fill over a circular or long-shape void (diameter D or width b, respectively).

5.1 Limit states

With reference to Figure 6, the limit states for design of a reinforced fill over a void are:
- Deformation serviceability limit state for the maximum allowable vertical deformation at the surface of the soil layer above the void (i.e., d_s < prescribed tolerable limit).
- Reinforcement strain serviceability limit state to keep the maximum strain (ε_{max}) in the reinforcement within a prescribed tolerable limit (ε_{allow}) (i.e., $\varepsilon_{max} < \varepsilon_{allow}$) or, equivalently, the value of d/b (or d/D) within a prescribed limit.
- Tensile strength limit state to ensure that the maximum computed tensile load (T_{max}) in the reinforcement does not exceed an allowable tensile capacity of the material.
- Stiffness serviceability limit state to ensure that the reinforcement has sufficient tensile stiffness to carry the maximum load in the reinforcement at the maximum tensile strain.

5.2 Reinforcement stiffness

A common feature of the deterministic and probabilistic analysis of the reinforced fill over a void problem investigated by writers is the treatment of the stiffness of the sheet reinforcement used to support the fill. Bathurst and Naftchali [9] developed a simple two-parameter hyperbolic isochronous load-strain model to characterize the stiffness of the reinforcement under constant load as a function of elapsed time and strain. This is an improvement over past practice that assumes that the reinforcement is a simple linear-elastic (strain-rate independent) material. They also compiled a large database of hyperbolic model parameters for five different types of geosynthetic sheet reinforcement products, and from these data generated typical model parameter ranges. They also developed correlations between creep stiffness for each type of reinforcement at an isochronous time of 1000 hours and different strain levels, and the ultimate tensile strength of these materials. These correlations are of practical value since creep data required to compute hyperbolic stiffness model parameters are not always available for project-specific sheet reinforcement materials at time of design.

5.3 Deterministic analyses

The analytical framework to compute maximum strains and load in the reinforcement is based on the methods from [6, 10-12]. Naftchali and Bathurst [13] demonstrated the conditions when the stiffness of the reinforcement will control design and when the strength of the reinforcement will control. They also showed that the common practice of using the stiffness from a conventional constant rate-of-strain test is non-conservative for design (i.e., the reinforcement stiffness is over-estimated). A practical outcome of this work is a design flow chart approach that considers the four limit states identified earlier. Example design charts for each of the four analytical methods found in the literature are also provided.

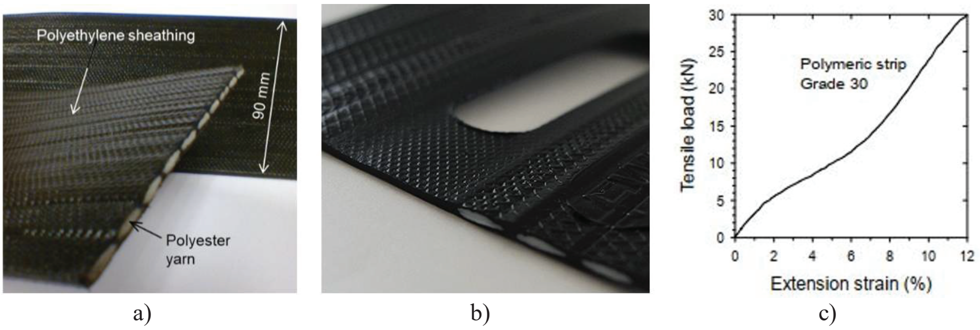
5.4 Probabilistic analyses

The same analytical model by Giroud et al. [10] was used by Bathurst and Naftchali [14] for probabilistic analyses of the reinforced fill over a void problem by assigning uncertainty to the estimate of reinforcement stiffness. This uncertainty was quantified from the results of creep testing in the database from [9]. They showed the quantitative link between the nominal factor of safety used in deterministic working stress design practice and reliability index. The latter is preferred in modern performance-based design to quantify margins of safety within a probabilistic framework. The presentation highlights the practical benefit of using product-specific isochronous secant stiffness data when available, rather than estimates of isochronous stiffness values based on reinforcement type or pooled data.

6 Numerical analysis of reinforced soil wall structures

There are currently many types of polymeric reinforcements available on the market, with different characteristics and limitations depending on their particular material composition and geometry. Among the multiple types of reinforced soil walls, this presentation mainly deals with those made up of concrete panels on the facing and polymeric strap-type reinforcement (Figure 7a). This type of reinforcement offers tensile strength up to 100 kN/strip in some products, and can be applied where metallic reinforcement has traditionally been considered. Polymeric straps usually have widths between 50 and 90 mm, and are made up of a polyester yarn core that provides tensile strength, and a polyethylene coating/sheath that provides protection and surface roughness/shear interaction with the surrounding soil. Currently there are other more or less novel solutions that try to improve/increase this interaction and adherence with the soil fill, for example, by means of an irregular geometry of the contours or with central holes (see Figure 7b).

There are several commercial software codes that can be used to model reinforced soil structures (e.g., PLAXIS [15], FLAC [16], CODE_BRIGHT [17], among others). Furthermore, numerical modeling is integrated as an optional design methodology for reinforced soil structures in the forthcoming Eurocode 7: EN 1997–Part 3 [18], Clause 9. It is important, then, to be able to determine the resistance and adequate stiffness for correct modeling of the reinforcing polymeric straps. Even though polyester is a significantly stiffer product compared to other types of polymeric materials, polyester strap reinforcement products exhibit relatively weak tensile behavior if their stress-strain behavior is analyzed based on their characteristic or short-term resistance, with a resulting extension of 10-12% regardless of the grade of the product (see Figure 7c). As shown, preliminary stiffness may be different if linear secant or tangent moduli is assumed due to the resulting S-shape load-extension trend. Furthermore, due to the application of partial reduction factors, this short-term/characteristic strength is typically reduced by less than 50% for ultimate limit states design purposes. If serviceability limit states can be prescribed with specific maximum strains of 1.0 or 0.5% (which depends on the type of structure and loading (as per Code BS 8006-1 [6]) it may be not trivial to directly assume the most accurate stiffness for modelling purposes (although this may not affect the final overall numerical results).



a) b) c)

Fig. 7. Polymeric strap details: (a) regular, (b) perforated/high adherence samples, and (c) typical load-extension behavior of Grade 30 strip [19, 20].

It is possible to analyze reinforced soil structures with numerical tools because, if properly calibrated/validated/verified, they provide accurate results with respect to the usual analytical methods (which can be significantly conservative under standardized conditions or according to traditional design standards (e.g., [6]). In addition, in this sense, since the use of numerical models is expected to be permitted in the next edition of Eurocode 7 [18] (also for reinforced soil structures), good use of these tools can lead to more refined optimization of the current systems, achieving tighter and more precise designs based on a more realistic performance of these types of structure. Even so, many details and specific analyses/studies are still pending to complement the current knowledge of reinforced soil structures (in particular, polymeric reinforcement) under extreme boundary conditions and alternative soil materials (such as recycled and marginal soils; cases with future applications based on climate change and the circular economy), in addition to extreme loading conditions (e.g., design for seismic and complex geometries, etc.). In this sense, when applied to geosynthetic reinforced soil structures, 2D (quite mature) and 3D (still novel) modeling can help to improve current system designs and propose interesting and appropriate improvements under sustainable criteria.

The scope and goals in which the numerical modeling of the reinforced soil wall can be useful is very wide [21-31]. Despite inefficient for regular/typical structures where analytical solutions are really fast and easy to use, numerical modelling can be used, as noted earlier, for design purposes [18]. However, numerical modelling can be very useful to analyze particular/non-regular projects and for global stability analyses (e.g., Figures 8a and 8b). Furthermore, to investigate specific component behavior of reinforced soil walls including their internal system interactions (e.g., soil-reinforcement interaction, horizontal panel joint behavior, back anchorage pre-tensioning, connection systems including concrete panel rebar, etc., e.g., Figures 8c and 8d), numerical modelling has proven to be a unique theoretical tool that, in combination with specific component tests, allows designer to expand the current state of design practice for reinforced soil structures.

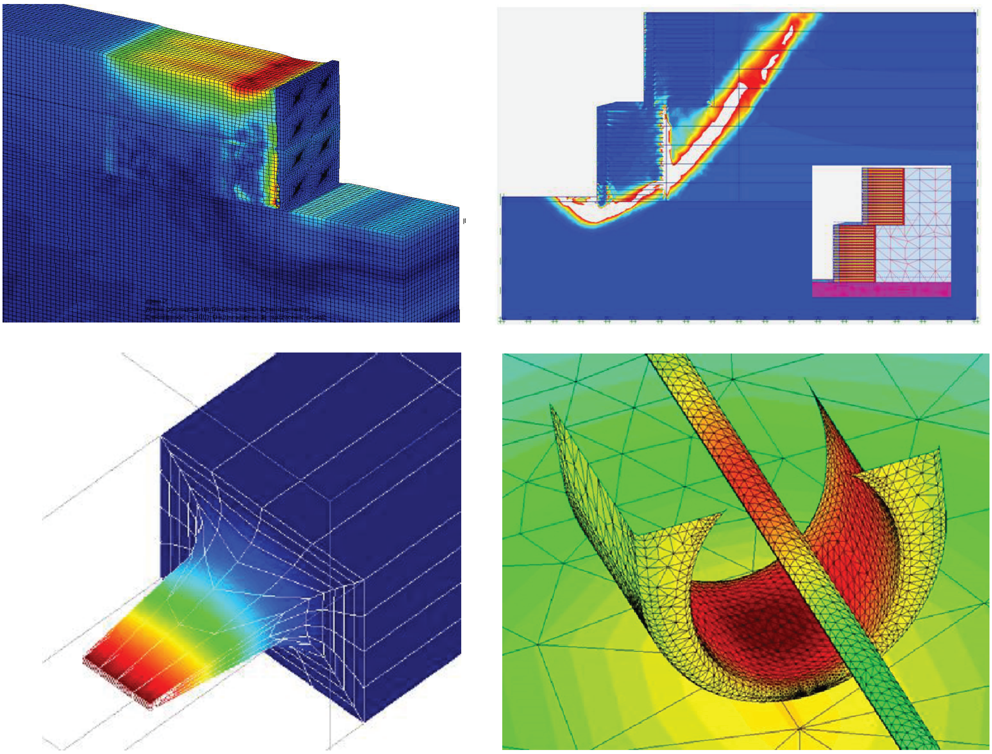


Fig. 8. Examples of numerical modelling outcomes from polymeric straps soil reinforcement applications: (a) CODE_BRIGHT [17] 3D modelling total displacement results for a low compacted wall case, (b) PLAXIS [15] 2D modelling with shear strains generated in a tiered wall case, and CODE_BRIGHT [17] 3D modelling of (c) soil-reinforcement pull-out tests and (d) void formed at panel connection pull-out tests.

The presentation addresses issues to be taken into account when designing with numerical modelling as well as special cases where the numerical modelling allows the designer to confirm mechanical performance suitability of polymeric strap reinforced soil wall alternatives.

7 Behavior and failure mechanisms of geosynthetic encased columns: Lessons learned from recent experiments

Geotechnical engineering practice has benefited from the use of granular columns in soft soil engineering since its inception in early 1970's. Classical literature on the subject accounts for the beneficial effects of granular column installation with the following list of desirable effects: reduction of total and differential settlements, reduction of drainage path (distance) and accelerated consolidation, increase in the overall stiffness and load bearing capacity of the soil, and stress relief in abutment walls. This list of beneficial effects is not exhaustive list but its prevalence in almost all the literature has to do with the widely accepted concept of the granular columns which is an infinite area of columns subjected to a uniform and vertically oriented stress field. Resistance to shear loads and increased resilience under dynamic loads is curiously not in the widely cited list of beneficial effects as these are not typically considered.

One of the earliest accounts of research on ordinary stone columns (OSCs) in geotechnical engineering practice can be traced back to Hughes et al. [32], where a field trial was undertaken to investigate the reinforcing effects of a singular stone column. Another pioneering study conducted in the same era (in fact, published in the same year as Hughes et al. [32]) was authored by Engelhardt and Golding [33], acknowledges that little if any information is available on the performance of granular column improved soils under seismic loading conditions [34]. Although the need to determine the seismic behavior of soft soil deposits enhanced by granular columns were accurately identified by early research, the progress in this particular area of geotechnical engineering did not match the widespread demand of the technique in soil remediation applications. In fact, the research interest on the behavior of the granular columns rarely extended beyond loading scenarios where static vertical loads were considered. With the widespread adaptation of the technique, it is evident that there are more load cases acting on the columns and column supported infrastructure assets such as road embankments, foundations, and bridge abutment walls. These load cases include lateral loads and moments, dynamic loads such as traffic loads, wave loads, and earthquake excitations. Given the importance of some of the infrastructure supported by these columns, it is crucial to understand the column and column supported soil behavior under the action of complex loading conditions.

7.1 Loads on granular columns – beyond static vertical loads

In addition to static vertical loads, the following loading patterns (or combinations thereof) might be imposed on the OSCs (ordinary stone columns) or GECs (geosynthetic enclosed columns):

- static/dynamic shear loads at a predefined shear plane;
- static/dynamic bending moments;
- cyclic or seismic loading.

While (in most cases) the primary function of the granular columns is to support the overlying superstructure in the vertical direction, the list of loading scenarios outlined above might cause the failure of the column and consequently the entire foundation system. Although the above-mentioned load cases might occur in combination, for the sake of simplicity, these will be elaborated in two main groups, namely, the behavior of columns under shear loads and the behavior of columns under dynamic/seismic loads.

7.1.1 Granular columns under shear loads

Shear failure is identified as the most common failure mechanism for sand compaction and stone columns [35]. There are several conceivable failure mechanisms which govern the behavior of OSCs and GECs which include (i) total shear failure of the column at the shear plane (with reinforcement rupture if the reinforcement exists and if the reinforcement stiffness is low); (ii) shearing of the encased column with the bending deformations around the shear plane; (iii) bending failure of the column. Shear failure occurs when the portion of the column above the slip circle is sufficiently restrained against lateral movement by the passive resistance of the circumferential unit cell soil around the column [36]. Mohapatra et al. [37] have further identified two sub-types of shear failure according to the behavior of the encasement in the vicinity of the shear plane. While GECs with a low reinforcement stiffness fail in rupturing of the reinforcement, columns encased with stiffer reinforcement materials are observed to fail in local bending of the column. The bending in this case occurs within the neighborhood of the shear plane passing through the unit cell and the column is laterally translated such that it maintains vertical disposition in the upper portion which is close to column head plane [36].

7.1.2 Granular columns under dynamic loads

The behavior of any element under dynamic loading conditions is dependent on the boundary conditions and the behavior of granular columns is not exempt from this physical rule. With the recent innovations in field applications which involve installation of GECs in various structural support roles the boundary conditions surrounding the column change drastically and column behavior should be studied in conjunction with the boundary conditions. Therefore, columns surrounded by free-field [38] conditions and columns in proximity to rigid boundary [39] conditions should be treated separately. The load carrying and settlement responses and the dynamic response of these columns will be different depending on the boundary conditions of the problem [40].

7.2 Observed behavior and conceivable failure modes of granular columns under complex loading conditions

Recent experimental research endeavors have shown the following highlights for granular columns and granular column supported clay beds subjected to complex loading patterns:
- An increase in the shear stiffness of the clay bed: in some cases the secant shear modulus of the soft soil was increased by an order of magnitude.
- A decrease in the lateral drift of the soil column (together with reduced shear strains) which means lower degradation of the shear modulus of the material under repeated loading cycles.
- Partial reinforcement of the column to mitigate the bulging failure of the granular column have gained some popularity in the academic research. The 1-g shaking table studies cited above have clearly shown that the seismically induced reinforcement strain demand is distributed along the height of the column. Therefore, partial reinforcement should be cautiously considered in seismically active sites.
- It is seen that the reinforcement strains under seismic loading conditions are closely related to the cumulative seismic energy exerted by the earthquake. In most cases, a linear transfer function can be derived to relate the seismic input energy to reinforcement strain.
- Similar to the point above, unit cell settlements can be predicted by making use of the seismic input energy.
- While the hoop strain is almost always considered as the governing factor for GECs, under shear loads, vertically oriented reinforcement strain is more pronounced and it can potentially cause a more onerous failure mechanism.

Interested readers are referred to the cited works of the author for detailed explanations of the behavioral patterns and failure mechanisms discussed above.

8 Load-carrying geosynthetic-reinforced bridge abutments

Geosynthetic-reinforced soil walls have been used extensively in transportation infrastructure worldwide. In the case of bridge abutments, this technology also been extensively used to support the loads induced by fill and pavement systems of approaching roads, while the load of the bridge itself has been often supported by transferring it to competent soil strata through deep foundations. The technology of geosynthetic-reinforced walls has evolved to directly support bridge loads, as illustrated in Figure 9 for the case of a bridge constructed in 1999 near Denver, Colorado, USA [41, 42]. This figure illustrates one of the configurations that has been adopted for load-carrying geosynthetic-reinforced (LCGR) bridge abutments, which is the focus of the given presentation.

The technology of LCGR bridge abutments has allowed avoiding the use of piles to transfer bridge superstructure loads to foundation soils and, consequently, alleviating the "bump at the end of bridge" (e.g., [43]) by allowing approaching roads and bridge decks to settle evenly under superstructure loads. Implementation of this alternative has also led to improvements in bridge construction. LCGR bridge abutments have been consistently reported to lead to cost-effective alternatives. For example, LCGR bridge abutments were reported to cost approximately 20 to 30 % less than alternative reinforced concrete abutments [44], and 25 to 60 % less than those constructed using traditional methods in general [43]. Furthermore, the use of LCGR bridge abutments reduces construction time when compared to other abutment alternatives, and thus minimizing traffic disruptions and shutdowns [43]. Recently, the US Federal Highway Administration (FHWA) has launched initiatives endorsing the use of geosynthetic-reinforced soils to construct bridge abutments as part of Accelerated-Bridge Construction (ABC) program that aim at addressing the problem of replacing deteriorated bridges with optimal mobility disruption, construction time, and cost. LCGR bridge abutments are also considered a more sustainable alternative than their conventional counterparts [45]. LCGR bridge abutments use 20 to 40 % less concrete than reinforced concrete abutments [44]. Additionally, LCGR abutments can be disassembled when used for temporary purposes, which allows recycling of both the reinforcement and fill [44].

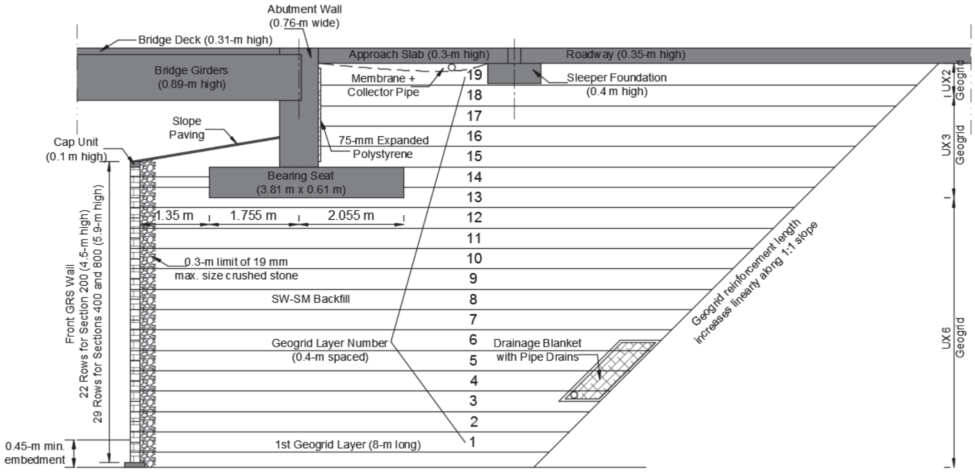


Fig. 9. Cross-section of the Founders/Meadows bridge, constructed near Denver, Colorado (redrawn from [42]).

8.1 Differences in Design Approach and Terminology

The philosophy adopted by different agencies worldwide for the design of LCGR bridge abutments may vary significantly, with one of the most significant differences being how to incorporate into the design the potential benefit of closely spaced reinforcement as well as the impact of the reinforcement stiffness. The importance of these variables was recognized early in the development of the unified design framework for soil reinforcement, although their impact has not necessarily been incorporated into several of the current design guidelines. For example, the early studies (e.g., [46, 47]) revealed the importance of accounting for the effect of reinforcement stiffness on the stresses within the reinforced soil mass, thereby incorporating appropriate relations into the design framework. The studies also established that wall face deformation, an important performance criterion, may be significantly affected by design factors not accounted for at that time, while noting that face deformation is highly sensitive to variations in the reinforcement vertical spacing. While a maximum reinforcement vertical spacing value was established to control face deformation, methods capitalizing on the additional advantages of closely spaced reinforcements were not identified at that time. Furthermore, most efforts at the time were devoted to the design of roadway walls, with only limited focus on bridge loads, where the use of closely spaced reinforcement was subsequently reported to result in significant performance advantages [43, 48].

The terminology adopted herein to refer to LCGR bridge abutments has varied widely in the technical literature, which may have obscured understanding the significance of different design approaches. For example, the terms "GR," "GRS," "MSE," and "GMSE" have been used since the 1980s, often indistinctly in the technical literature, to refer to retaining structures that are reinforced with geosynthetics. While these various terms have been used generically and irrespective of their reinforcement vertical spacing in most of the technical literature, it is recognized that the term "GRS" has been associated with small reinforcement vertical spacing in some FHWA guidelines [43, 49]. Geosynthetic-reinforced walls constructed as part of a bridge system can be identified as "geosynthetic-reinforced (GR) bridge abutments," irrespective of whether they carry only the load of the approaching road (i.e., the most common case) or the loads of both the approaching road and the bridge superstructure.

The more specific term "load-carrying geosynthetic-reinforced (LCGR) bridge abutment" is adopted herein to identify GR bridge abutments whose reinforced fill receive the full load of the bridge superstructure, rather than transferring such load directly to the foundation soils via deep foundation systems. It should be noted, however, that other terms (e.g., "true abutments") have been used to describe this type of system. While the term "LCGR bridge abutment" applies to any reinforcement vertical spacing, the term "geosynthetic-reinforced soil—integrated bridge abutment system (GRS-IBS)" has been used for structures designed following FHWA guidelines [43, 49] that prescribe not only a comparatively small reinforcement vertical spacing, but also rather specific requirements for construction and materials. Accordingly, GRS-IBS structures are identified as a subset of the more generic "LCGR bridge abutment" systems.

The term "integral" abutment has been used by bridge engineers to identify those abutments that have: (1) no thermal expansion joints between the bridge superstructure and approach road; and (2) no bearings or elastomeric pads isolating the superstructure from the substructure [50]. Some GMSE bridge abutments, including most GRS-IBS structures, would classify as "integral" bridges according to this definition even though the GRS-IBS structures may not necessarily involve integration between the GR abutments, the bridge superstructure, and the approaching roads. Figure 10 summarizes the interrelationship among the terms "GR walls," "GR bridge abutments," "load-carrying GR bridge abutments," "integral GR bridge abutments," and "GRS-IBS."

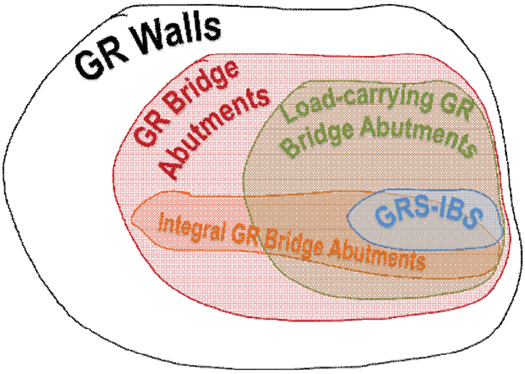


Fig. 10. Interrelation of different GR structures (based on [48]).

References

1. ISO 10318-1, *Geosynthetics − Part 1: Terms and definitions.* ISO, Geneve, Switzerland (2015).
2. ISO/TR 18228-1, *Design using geosynthetics − Part 1: General.* ISO, Geneve, Switzerland (2021).
3. N. Dixon, G. Fowmes, M. Frost, *Global challenges, geosynthetic solutions and counting carbon*, ICE Geos. Int., **24**(5), pp 451-464 (2017).
4. N. Touze, *Healing the world: A geosynthetics solution*, ICE Geos. Int., **28**(1), pp. 1-31 (2021).
5. EN 14475, *Execution of special geotechnical Works - Reinforced fill*, European Standard EN 14475:2006. CEN, Brussels, Belgium (2006).
6. BS 8006-1. *Code of practice for strengthened/reinforced soils and other fills* (+A1:2016), BSI, UK. (2010).
7. VSL Construction systems, *VSoL® System*, VSL International – a member of Bouygues construction, 08908 L'Hospitalet de Llobregat, Barcelona, Spain.
8. Officine Maccaferri SpA, Bologna, Italy.
9. R.J. Bathurst, F.M. Naftchali, *Geosynthetic reinforcement stiffness for analytical and numerical modelling of reinforced soil structures*, Geot. & Geom., **49**(4), pp. 921-940 (2021).
10. J.P. Giroud, R. Bonaparte, J.F. Beech, B.A. Gross, *Design of soil layer-geosynthetic systems overlying voids*, Geot. & Geom., **9**(1) pp. 11-50 (1990).
11. J.C. Blivet, J.P. Gourc, P. Villard, H. Giraud, M. Khay, A. Morbois, *Design method for geosynthetic as reinforcement for embankment subjected to localized subsidence*, in Proc. of the 7th Int. Conf. on Geosynthetics – Vol. 1, pp. 341-344, Lisse, France (2002).
12. EBGEO, *Recommendations for design and analysis of earth structures using geosynthetic reinforcements*, DGGT, Essen, Germany (2011).
13. F.M Naftchali, R.J. Bathurst, *Influence of geosynthetic stiffness on analytical solutions for reinforced fill over void*, ICE Geos. Int., (Ahead of Print) (2022).
14. R.J. Bathurst, F.M. Naftchali, *Influence of uncertainty in geosynthetic stiffness on deterministic and probabilistic analyses using analytical solutions for three reinforced soil problem*, Geot. & Geom., **51**(1), pp. 117-130 (2023).
15. PLAXIS, *PLAXIS 2D Reference Manual*, Delft University of Technology, Delft, Netherlands.
16. FLAC, *Fast Lagrangian Analysis of Continua (FLAC) - User's Guide*, Itasca Consulting Group, Inc., Minneapolis, USA.
17. S. Olivella, A. Gens, J. Carrera, E.E. Alonso, *Numerical Formulation for a Simulator (CODE_BRIGHT) for the Coupled Analysis of Saline Media*, Eng. Comp., **13**(7), pp. 87-112 (1996).
18. EN 1997-3, *Geotechnical design — Part 3: Geotechnical structures*, European Standard EN 1997, CEN/TC 250, Brussels, Belgium (202x - under development).
19. GECO Industrial Co. Ltd., *Polymeric reinforcing strip FASTEN®*, Jangan-myeon, Gyeonggi-do, 18580, Republic of Korea.
20. A. Brouthen, M.N. Houhou, I.P. Damians, *Numerical study of the influence of the interaction distance, the polymeric strips pre-tensioning, and the soil-polymeric interaction on the performance of back-to-back reinforced soil walls*, Infr. **7**(2), (2022).

21. I.P. Damians, R.J. Bathurst, A. Josa, A. Lloret, P.J.R. Albuquerque, *Vertical facing loads in steel reinforced soil walls*. ASCE J. of Geot. and Geoenv. Eng. **139**(9), pp. 1419-1432 (2013).
22. I.P. Damians, R.J. Bathurst, A. Josa, A. Lloret, *Numerical study of the influence of foundation compressibility and reinforcement stiffness on the behavior of reinforced soil walls*. Int. J. of Geot. Eng., **8**(3), pp. 247-259 (2014).
23. Y. Yu, I.P. Damians, R.J. Bathurst, *Influence of choice of FLAC and PLAXIS interface models on reinforced soil-structure interactions*, Comp. & Geotech., **65**, pp. 164-174 (2015).
24. I.P. Damians, R.J. Bathurst, A. Josa, A. Lloret, *Numerical analysis of an instrumented steel reinforced soil wall*. ASCE Int. J. of Geom., **15**(1), (2015).
25. I.P. Damians, R.J. Bathurst, A. Josa, A. Lloret, *Vertical facing panel-joint gap analysis for steel-reinforced soil walls*. ASCE Int. J. of Geom., **16**(4), (2016).
26. I.P. Damians, R.J. Bathurst, E. Adroguer, A. Josa, A. Lloret, *Environmental assessment of earth retaining wall structures*. ICE Env. Geotech., **4**(6), pp. 415-431 (2017).
27. I.P. Damians, R.J. Bathurst, E. Adroguer, A. Josa, A. Lloret, *Sustainability assessment of earth retaining wall structures*. ICE Env. Geotech., **5**(4), pp. 187-203 (2018).
28. I.P. Damians, R.J. Bathurst, S. Olivella, A. Josa, A. Lloret, *3D modelling of strip reinforced MSE walls*, Acta Geotech. **16**, pp. 711-730 (2021).
29. I.P. Damians, S. Olivella, R.J. Bathurst, A. Lloret, A. Josa, *Modelling soil-facing interface interaction with continuum element methodology*, Frontiers in Built Environment – Insights in Geotech. Eng. (2022).
30. A.A. Moncada, I.P. Damians, S. Olivella, R.J. Bathurst, *Modelling the effect of in-soil temperature and relative humidity on performance of PET strap soil reinforcement products*, in Proc. of the 20th Int. Conf. on Soil Mech. and Geot. Eng., Sydney, Australia (2022).
31. E. Gil, C. Serrano, C. Pereira, P. Osso, J., Lima, I.P. Damians, *The D4R7 reinforced soil retaining walls in Bratislava, Slovakia*, Int. J. of Geoeng. Case Histories, **7**(2), pp. 13-33 (2022).
32. J. Hughes, N. Withers, D. Greenwood, *A field trial of the reinforcing effect of a stone column in soil*. Geotechnique, **25**(1), pp. 31-44 (1975).
33. K. Engelhardt, H. Golding, *Field testing to evaluate stone column performance in a seismic area*, Geotechnique, **25**(1), pp. 61-69 (1975).
34. C. Cengiz, E. Guler, *Effects of geosynthetic encased column installation on the seismic response of soft clay beds*, in Proc. of the 7th Eur. Geos. Conf., EuroGeo7, Warsaw, Poland (2021).
35. S.W. Abusharar, J. Han, *Two-dimensional deep-seated slope stability analysis of embankments over stone column-improved soft clay*, Eng. Geol., **120**(1), pp. 103-110 (2011).
36. C. Cengiz, I.E. Kilic, E. Guler, *On the shear failure mode of granular column embedded unit cells subjected to static and cyclic shear loads*, Geot. & Geom., **47**(2), pp. 193-202 (2019).
37. S.R. Mohapatra, K. Rajagopal, J. Sharma, *Direct shear tests on geosynthetic-encased granular columns*, Geot. & Geom., **44**(3), pp. 396-405 (2016).
38. C. Cengiz, E. Guler, *Shaking table tests on geosynthetic encased columns in soft clay*, Geot. & Geom., **46**(6), pp. 748-758 (2018).

39. C. Cengiz, E. Güler, *Seismic behavior of geosynthetic encased columns and ordinary stone columns*, Geot. & Geom., **46**(1), pp. 40-51 (2018).
40. Cengiz, C. and E. Guler, Load bearing and settlement characteristics of Geosynthetic Encased Columns under seismic loads. Soil Dynamics and Earthquake Engineering, 2020. 136: p. 106244.
41. J.G. Zornberg, N. Abu-Hejleh, T. Wang, *Geosynthetic-reinforced soil bridge abutments*, Geot. Fabrics Rep., **19**(2), pp. 52-55 (2001).
42. N. Abu-Hejleh, J.G. Zornberg, T. Wang, J. Watcharamonthein, *Monitored displacements of unique geosynthetic-reinforced soil bridge abutments*, ICE Geos. Int., **9**(1), pp. 71-95 (2002).
43. M. Adams, J. Nicks, T. Stabile, J. Wu, W. Schlatter, J. Hartmann, *Geosynthetic-reinforced soil integrated bridge system,* FHWA-HRT-11-027, McLean, VA. (2011).
44. A. Herold, *Brückenwiderlager aus KBE-Kunststoffbewehrte Erde, Einsatzgebiete Und Anwendungsgrenzen*, Sächsisches Textilforschungsinstitut Bautextilien-Symposium, Bautex 2006, Institut für Technische Textilien GmbH, Chemnitz, pp. 1-12 (2006).
45. K.F. Bizjak, S. Lenart, S., *Life cycle assessment of a geosynthetic-reinforced soil bridge system–A case study*, Geot. & Geom., **46**(5), 543-558 (2018).
46. J.K. Mitchell, W.C. Villet, *Reinforcement of earth slopes and embankments*, NCHRP Report (290), Transp. Res. Board, Washington, D.C. (1987).
47. B.R. Christopher, S. Gill, J. Giroud, I. Juran, J.K. Mitchell, F. Schlosser, J. Dunnicliff, *Reinforced soil structures. Design and construction guidelines – Vol. 1*, FHWA-RD-89-043, Washington, D.C. (1989).
48. J.G. Zornberg, A.M. Morsy, B. Mofarraj, B.R. Christopher, D. Leshchinsky, J. Han, B.F. Tanyu, F.T. Gebremariam, P. Shen, Y. Jiang, *Defining the boundary conditions for composite behavior of geosynthetic reinforced soil (GRS) structures*, NCHRP, Project 24-41, Transport. Res. Board, Washington DC, October, 986 pp. (2018).
49. M. Adams, J. Nicks, *Design and construction guidelines for geosynthetic reinforced abutments and integrated bridge systems*, FHWA-HRT-17-080, McLean, VA. (2018).
50. M.P. Burke, *Integral and semi-integral bridges*, Wiley-Blackwell, Chichester, U.K., Ames, Iowa (2009).